Equations of Straight Lines 2

1.

The lines $y = \frac{a}{3}x - 4$ and $y = 3 - \frac{b}{4}x$ are perpendicular.

Find the value of ab.

Circle your answer.

[1 mark]

$\frac{3}{4}$ $\quad$ -12 $\quad$ $-\frac{4}{3}$ $\quad$ 12

2.

Determine whether the line with equation $2x + 3y + 4 = 0$ is parallel to the line through the points with coordinates (9, 4) and (3, 8).

[4 marks]

3.

The points *A* and *B* have coordinates (1, –2) and (5, 6) respectively.

Given that the point with coordinates $(p, p + 8)$ lies on the perpendicular bisector of *AB*, find the value of p.

[4 marks]

4.

Point *C* has coordinates (c, 2) and point *D* has coordinates (6, d).

The line $y + 4x = 11$ is the perpendicular bisector of *CD*.

Find c and d.

[5 marks]

5.

Points *A* (–7, –7), *B* (8, –1), *C* (4, 9) and *D* (–11, 3) are the vertices of a quadrilateral *ABCD*.

(a) Prove that *ABCD* is a rectangle.

[4 marks]

(b) Find the area of *ABCD*.

[2 marks]

6.

The straight line l has a gradient of $-\frac{5}{12}$, and passes through the points $A(10,1)$ and $B(k,11)$, where k is a constant.

a) Find an equation of l, in the form $ax+by=c$, where a, b and c are integers.

b) Determine the value of k.

c) Hence show that the distance AB is 26 units.

7.

The straight line L passes through the points $(2,5)$ and $(-2,3)$, and meets the coordinate axes at the points P and Q.

Find the area of a square whose side is PQ.

8.

The straight line l_1 passes through the points $A(-1,-1)$ and $B(k,5)$, where k is a constant.

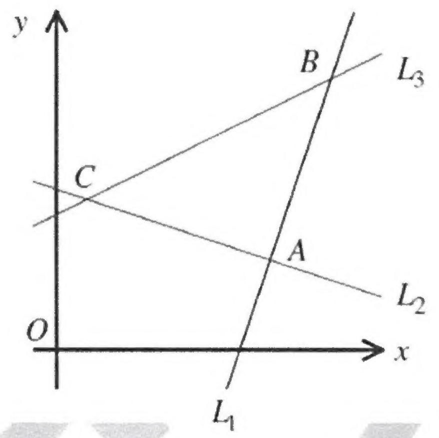
a) Given that the gradient of l_1 is $\frac{1}{2}$ show that $k=11$.

The straight line l_2 passes through the midpoint of AB and is perpendicular to l_1.

b) Determine an equation of l_2, giving the answer in the form $ax+by=c$, where a, b and c are integers.

c) Calculate the area of the triangle enclosed by l_2 and the coordinate axes.

9.



The figure above shows three straight lines L_1, L_2 and L_3.

a) Find an equation of the straight line L_1, given that it passes through the points $A(7,3)$ and $B(9,9)$.

L_2 is perpendicular to L_1 and passes through A.

b) Find an equation of L_2.

L_3 meets L_1 at the point B and L_2 at the point C.

The equation of L_3 is $y = \frac{x+9}{2}$.

c) Determine the coordinates of C.

d) Show that the triangle ABC is isosceles.

10.
The points A and B have coordinates $(3,-5)$ and $B(1,1)$, respectively.

a) Find an equation of the straight line through A and B, in the form $ax+by=c$ where a, b and c are integers.

The midpoint of AB is M, and the line segment MC is perpendicular to AB.

b) Find the coordinates of M.

c) State the gradient of MC.

d) Given that C has coordinates $(8,a)$, find the value of a.

11.
The points $A(0,3)$, $B(2,-1)$ and $C(k,1)$ are given, where k is a constant.

a) Find the exact length of AB.

b) Given that AB is perpendicular to BC, find the value of k.

c) Determine the area of the triangle ABC

12.
The points A and B have coordinates $(1,4\sqrt{3})$ and $(-3+\sqrt{3},3)$, respectively.

a) Show that the gradient of AB is $\sqrt{3}$.

b) Find an equation for the straight line L which passes through A and B.

L meets the x axis at the point C.

c) Determine the length of AC.

d) Calculate the acute angle between L and the x axis.

Equations of Straight Lines 2 MS

1.

Circles correct answer	AO1.1b	B1	12
Total		**1**	

2.

Explains that equal gradients implies that lines are parallel	AO2.4	E1	Parallel lines have equal gradient
Finds the gradient of the given line CAO	AO1.1b	B1	$2x + 3y + 4 = 0 \Rightarrow y = -\frac{2}{3}x - \frac{4}{3}$ So gradient is $-\frac{2}{3}$
Finds the gradient of the line through the 2 given points CAO	AO1.1b	B1	Gradient of line through (9, 4) and (3, 8) is $\frac{8-4}{3-9} = -\frac{2}{3}$
Deduces that the two lines are parallel	AO2.2a	R1	So line with equation $2x + 3y + 4 = 0$ is parallel to the line joining the points with coordinates (9, 4) and (3, 8) as both have gradient $-\frac{2}{3}$
Total		**4**	

3.

Selects an appropriate method by finding the midpoint of *AB* and the gradient of *AB*	AO3.1a	M1	Mid-point of $AB = (3, 2)$ Gradient of $AB = 2$ Hence gradient of perpendicular bisector $= -\frac{1}{2}$ Equation of perpendicular bisector is $y - 2 = -\frac{1}{2}(x - 3)$ $p + 6 = -\frac{1}{2}(p - 3)$ $p = -3$
Finds the correct gradient of the perpendicular bisector of *AB* ft 'their' gradient of *AB*	AO1.1b	A1F	
Forms an appropriate equation and substitutes the given coordinate into 'their' equation to find p	AO1.1a	M1	
Finds the correct value of p	AO1.1b	A1	
Total		**4**	

4.

Forms an equation for gradient of CD = ¼ or –¼ of the form difference in y over difference in x (or vice versa = 4 or -4)	AO3.1a	M1	$\frac{d-2}{6-c}=\frac{1}{4}$ $4d-8=6-c$
Obtains a correct equation for c & d	AO1.1b	A1	$c+4d=14$
Forms an equation for the mid-point of CD lying on $y+4x=11$	AO3.1a	M1	$\frac{2+d}{2}+4\left(\frac{c+6}{2}\right)=11$
Obtains correct equation for c & d (any correct form)	AO1.1b	A1	$4c+d=-4$
Solves for c and d CAO	AO1.1b	A1	$c=-2 \quad d=4$
Total		**5**	

5.

(a)	Selects a method leading to any calculation pertaining to one of the following methods seen (not necessarily correct); gradients of sides, lengths of sides or intersection or lengths of diagonals	AO3.1a	M1	Grad BC = -5/2 = Grad DA Grad AB = 2/5 = Grad DC Both pairs of opposite sides have equal gradient so parallel, so ABCD is a parallelogram
	Finds gradients of all 4 sides or lengths of all 4 sides or midpoints of both diagonals correctly	AO1.1b	A1	
	Proves one angle is 90° by using gradients or Pythagoras	AO1.1a	M1	Grad BC × grad AB = -1 ABC = 90° therefore all angles in ABCD are 90° so ABCD is a rectangle
	Completes proof that ABCD is a rectangle. There must be a clear statement that there are 2 pairs of parallel sides and all angles are 90°	AO2.1	R1	
	Note, there are various ways of proving that ABCD is a rectangle (1 – 5 below score M1 A1 M1 before final required statement for relevant R1 stating how their method used proves a rectangle) 1. As in the typical solution shown: show that both pairs of opposite sides are parallel, show that one angle is 90°. 2. Show that each pair of opposite sides is equal in length, show that one angle is 90°. 3. Show that one pair of opposite sides is parallel and equal in length, show that one angle is 90°. 4. Show that the diagonals bisect (the midpoint of one is also the midpoint of the other) and are equal in length. 5. Show that each pair of opposite sides are parallel and length of the two diagonals are the same NB May be expressed using vectors NB Diagonals AC and BD = $\sqrt{377}$			
b)	Finds correct lengths of two adjacent sides (accept to at least 1dp accuracy)	AO1.1a	M1	AB (= DC) = $\sqrt{261}=3\sqrt{29}$ BC (= DA) = $\sqrt{116}=2\sqrt{29}$
	Obtains correct area (AWRT)	AO1.1b	A1	Area = 174
	Total		**6**	

6.

(a) $y - y_0 = m(x - x_0)$

$y - 1 = -\frac{5}{12}(x - 10)$

$12y - 12 = -5x + 50$

$12y + 5x = 62$

(b) $B(k, 11)$ LIES ON THE LINE

$12 \times 11 + 5k = 62$

$132 + 5k = 62$

$5k = -70$

$k = -14$

(c) $|AB| = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

$|AB| = \sqrt{(11 - 1)^2 + (-14 - 10)^2}$

$|AB| = \sqrt{10^2 + (-24)^2}$

$|AB| = \sqrt{100 + 576}$

$|AB| = \sqrt{676} = 26$ AS REQUIRED

7.

- GRAD $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 5}{-2 - 2} = \frac{-2}{-4} = \frac{1}{2}$

- $y - y_0 = m(x - x_0)$ WITH $m = \frac{1}{2}$ @ $(2, 5)$

  $y - 5 = \frac{1}{2}(x - 2)$

  $2y - 10 = x - 2$

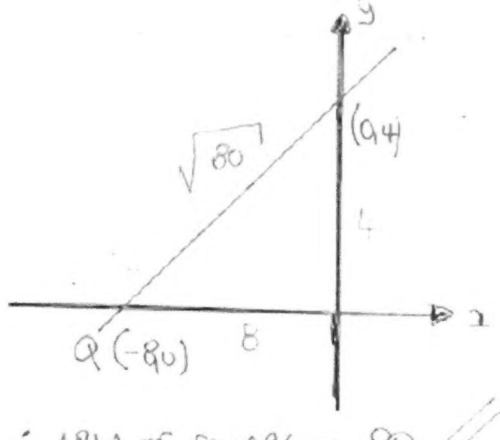
  $\boxed{2y = x + 8}$

- WHEN $x = 0$, $y = 0$

  $2y = 8$ $\quad$ $0 = x + 8$

  $y = 4$ $\quad$ $x = -8$

  $\therefore P(0, 4)$ $\quad$ $Q(-8, 0)$



$\therefore$ AREA OF SQUARE $= 80$

8.

(a) $m = \frac{1}{2}$

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{2}$

$\frac{5-(-1)}{k-(-1)} = \frac{1}{2}$

$\frac{6}{k+1} = \frac{1}{2}$

$k+1 = 12$

$k = 11$

(b) MIDPOINT $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$M\left(\frac{-1+11}{2}, \frac{-1+5}{2}\right)$

$M(5, 2)$

$y - y_0 = m(x - x_0)$ WITH $m = -2$

$y - 2 = -2(x-5)$

$y - 2 = -2x + 10$

$y = 12 - 2x$

$y + 2x = 12$

(c) WITH $x = 0$

$y = 12$

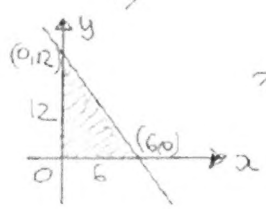
$\therefore (0,12)$

& $y = 0$

$2x = 12$

$x = 6$

$\therefore (6,0)$



AREA $= \frac{1}{2} \times 6 \times 12$

$= 36$

9.

(a) GRADIENT AB $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-3}{9-7} = \frac{6}{2} = 3$

$\therefore L_1$: $y - y_1 = m(x - x_1)$

$y - 9 = 3(x-9)$

$y - 9 = 3x - 27$

$y = 3x - 18$

(b) GRADIENT OF L_2 MUST BE $-\frac{1}{3}$ & PASSING THROUGH $A(7,3)$

$y - y_0 = m(x - x_0)$

$y - 3 = -\frac{1}{3}(x-7)$

$3y - 9 = -x + 7$

$3y + x = 16$

(c) $L_2: \; 3y + x = 16$
$L_3: \; y = \dfrac{x+9}{2}$

SOLVING SIMULTANEOUSLY $\Rightarrow 3\left(\dfrac{x+9}{2}\right) + x = 16$

$$3(x+9) + 2x = 32$$
$$3x + 27 + 2x = 32$$
$$5x = 5$$
$$x = 1$$
$$\& \; y = 5$$
$$\therefore C(1,5)$$

(d) IT SUFFICES TO CHECK THE LENGTHS OF $|AC|$ & $|AB|$ AS THERE IS A RIGHT ANGLE AT A

$$|AC| = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2} = \sqrt{(5-3)^2 + (1-7)^2} = \sqrt{4+36} = \sqrt{40}$$
$$|AB| = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2} = \sqrt{(9-3)^2 + (9-7)^2} = \sqrt{36+4} = \sqrt{40}$$

$$|AB| = |AC| \neq |BC|.$$

INDEED ISOSCELES

10.

(a) GRADIENT $AB = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{1+5}{1-3} = \dfrac{6}{-2} = -3$

SO $m = -3$ & $B(1,1)$

$$y - y_0 = m(x - x_0)$$
$$y - 1 = -3(x-1)$$
$$y - 1 = -3x + 3$$
$$3x + y = 4$$

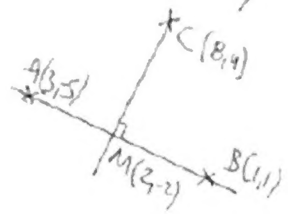
(b) MIDPOINT OF AB IS $M\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$\therefore M\left(\dfrac{3+1}{2}, \dfrac{-5+1}{2}\right) \Rightarrow M(2,-2)$

(c) $MC \perp AB$

$\therefore$ GRADIENT $MC = \dfrac{1}{3}$

(d)



GRADIENT $MC = \dfrac{1}{3}$

$$\dfrac{a+2}{8-2} = \dfrac{1}{3}$$
$$\dfrac{a+2}{6} = \dfrac{1}{3}$$

$$a + 2 = 2$$
$$a = 0$$

11.

(a) $|AB| = \sqrt{(y_2-y_1)^2-(x_2-x_1)^2} = \sqrt{(-1-3)^2+(2-0)^2} = \sqrt{16+4} = \sqrt{20}$

(b) GRADIENT AB $= \frac{y_2-y_1}{x_2-x_1} = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$

GRADIENT BC $= \frac{1-(-1)}{k-2} = \frac{2}{k-2}$

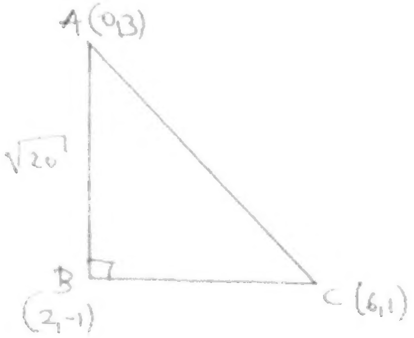
$\Rightarrow$ GRADIENTS ARE NEGATIVE RECIPROCALS

if $\frac{2}{k-2} = \frac{1}{2}$

$\Rightarrow k-2=4$

$\Rightarrow k=6$

(c)



$|BC| = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$= \sqrt{(1+1)^2+(6-2)^2} = \sqrt{4+16}$

$= \sqrt{20}$

$\therefore$ AREA $= \frac{1}{2}|AB||BC|$

$= \frac{1}{2}\sqrt{20}\sqrt{20}$

$= 10$

12.

(a) GRADIENT AB $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 4\sqrt{3}}{-3+\sqrt{3}-1} = \frac{3-4\sqrt{3}}{-4+\sqrt{3}}$

$= \frac{(3-4\sqrt{3})(-4-\sqrt{3})}{(-4+\sqrt{3})(-4-\sqrt{3})} = \frac{\cancel{-12} - 3\sqrt{3} + 16\sqrt{3} \cancel{+12}}{16 \cancel{+4\sqrt{3}} \cancel{-4\sqrt{3}} - 3}$

$= \frac{13\sqrt{3}}{13} = \sqrt{3}$

(b) USING $A(1, 4\sqrt{3})$ & $m = \sqrt{3}$

$y - y_0 = m(x - x_0)$

$y - 4\sqrt{3} = \sqrt{3}(x-1)$

$y - 4\sqrt{3} = \sqrt{3}x - \sqrt{3}$

$y = \sqrt{3}x + 3\sqrt{3}$

(c) when $y = 0$

$0 = \sqrt{3}x + 3\sqrt{3}$

$-3\sqrt{3} = \sqrt{3}x$

$-3 = x$

$\therefore C(-3,0)$ & $A(1, 4\sqrt{3})$

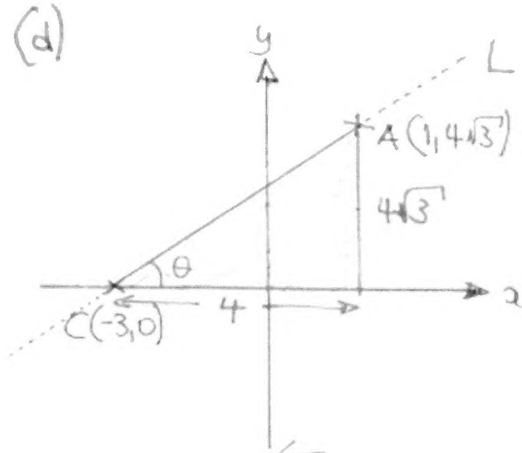
$d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$

$|AC| = \sqrt{(4\sqrt{3})^2 + (1+3)^2}$

$|AC| = \sqrt{48+16}$

$|AC| = 8$

(d)



$\tan\theta = \frac{\cancel{4}\sqrt{3}}{\cancel{4}}$

$\tan\theta = \sqrt{3}$

$\theta = 60°$